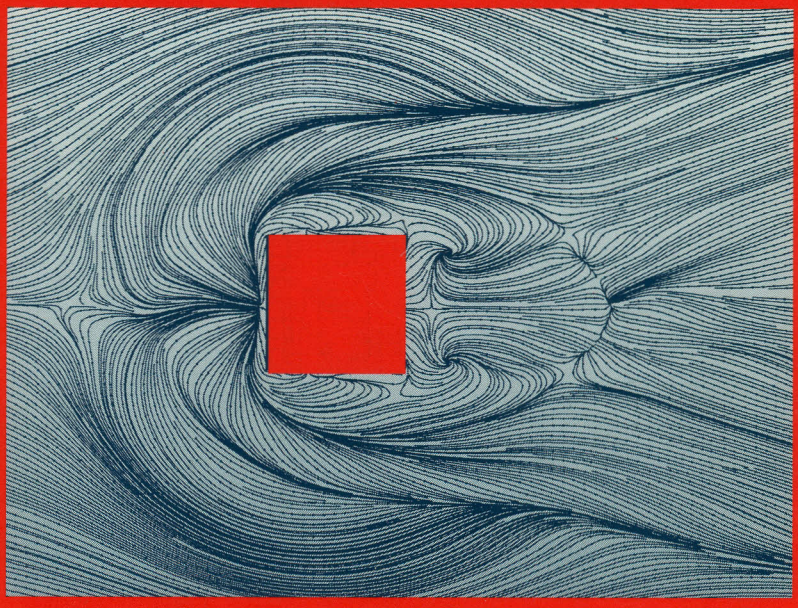
Joel H. Ferziger · Milovan Perić

Computational Methods for Fluid Dynamics

Second Edition



Springer

J. H. Ferziger · M. Perić

Computational Methods for Fluid Dynamics

Second Revised Edition with 118 Figures

D III 272
DK: 551.511.3
551.501.4

339/4120 INSTITUT
FÜR METEOROLOGIE U. KLIMATOLOGIE
UNIVERSITÄT HANNOVER
HERRENHÄUSER STR. 2 · 30419 HANNOVER

Springer

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